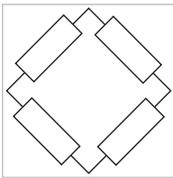


Applying the Wheatstone Bridge Circuit

by Karl Hoffmann



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1 Introduction

In 1843 the English physicist, Sir Charles Wheatstone (1802-1875), found a bridge circuit for measuring electrical resistances. In this bridge circuit, known today as the Wheatstone bridge circuit, unknown resistances are compared with well-defined resistances [1]. The Wheatstone bridge is also well suited for the measurement of small changes of a resistance and is, therefore, also suitable for measuring the resistance change in a strain gage (SG). It is commonly known that the strain gage transforms strain applied into a proportional change of resistance. The relationship between the applied strain ε ($\varepsilon = \Delta L/L_0$) and the relative change of the resistance of a strain gage is described by the equation

$$\frac{\Delta R}{R_0} = k \cdot \varepsilon \quad (1)$$

The factor k , also known as the gage factor, is a characteristic of the strain gage and has been checked experimentally [2]. The exact value is specified on each strain gage package. In general, the gage factor for metal strain gages is about 2.

Below, the Wheatstone bridge circuit will only be considered with respect to its application in strain gage technique. Two different presentations are given in fig. 1: a) is based on the original notation of Wheatstone, and b) is another notation that is usually easier to understand by a person without a background in electrical or electronic engineering. Both versions are, in fact, identical in their electrical function.

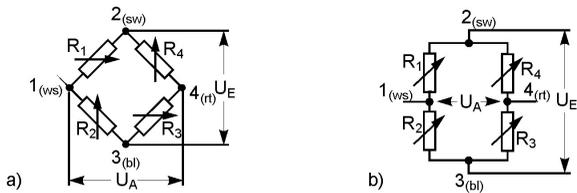


Fig. 1: The Wheatstone bridge circuit

The four arms (or branches) of the bridge are formed by the resistors R_1 to R_4 . The corner points (or nodes) are numbered and color-coded according to the HBM standard for designating the connections of transducers and instruments. If nodes 2 (black) and 3 (blue) - the so-called excitation diagonal - are connected to a known voltage U_E (bridge input or excitation voltage) then a voltage U_A (bridge output voltage) appears between nodes 1 (white) and 4 (red), the so-called measurement diagonal. The value of the output voltage depends on the ratio of the resistors $R_1 : R_2$ and $R_4 : R_3$.

Note 1-1: For the sake of clarity, it is more convenient to consider the ratio of the output and the input voltage U_A / U_E . Therefore, the following equations are set up for this ratio.

Generally the equation

$$\frac{U_A}{U_E} = \frac{R_1}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} = \frac{R_1 \cdot R_3 - R_2 \cdot R_4}{(R_1 + R_2)(R_3 + R_4)} \quad (2)$$

is valid and for the case of the balanced bridge we have

$$\frac{U_A}{U_E} = 0 \quad \text{if } R_1 = R_2 = R_3 = R_4 \quad (3)$$

$$\text{or } R_1 : R_2 = R_4 : R_3$$

Note 1-2: In all practical cases where strain gages are used, the balanced state is only reached with more or less deviation. Therefore, all instruments designed for strain gage measurements are equipped with balancing elements, which allow the indication to be set to zero for the initial state. This allows the use of equation (3) for all further considerations.

Another assumption is that the bridge output remains unloaded electrically. The internal resistance of the instrument connected to this output must be large enough to prevent noticeable errors. This is assumed for all further considerations.

If the resistors R_1 to R_4 vary, the bridge will be detuned and an output voltage U_A will appear. With the additional assumption that the resistance variation ΔR_i is much smaller than the resistance R_i itself (which is always true for metal strain gages), second order factors can be disregarded. We now have the following relationship

$$\frac{U_A}{U_E} = \frac{1}{4} \left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4} \right) \quad (4)$$

Note 1-3: For practical strain gage applications, the pairs R_1, R_2 and R_3, R_4 or all four resistors R_1 to R_4 should have the same nominal value to ensure that the relative changes of the individual bridge arms are proportional to the relative variation of the output voltage. It is not significant whether R_1 and R_4 (or R_2 and R_3) have the same or a different nominal resistance. This is the reason for always assuming $R_1 = R_2 = R_3 = R_4$.

Substituting equation (1) ($\Delta R/R_0 = k \cdot \epsilon$) in (4) we find

$$\frac{U_A}{U_E} = \frac{k}{4} \cdot (\epsilon_1 - \epsilon_2 + \epsilon_3 - \epsilon_4) \quad (5)$$

The signs of the terms are defined as follows (see also fig. 2):

With the given polarity of the excitation voltage U_E : node (2) = negative, (3) = positive, we will have

positive potential at point (1), negative potential at point (4), if $R_1 > R_2$ and/or $R_3 > R_4$;

negative potential at (1), positive potential at (4), if $R_1 < R_2$ and/or $R_3 < R_4$.

For a.c. supply, the above conditions are true for the phase relations of U_E and U_A .

Please note: The changes of the absolute values of **neighboring** strain gages are **subtracted if they have like signs**. They are **added if they have unlike signs**. This fact can be used for some combinations or compensation methods, which will be discussed in detail separately.

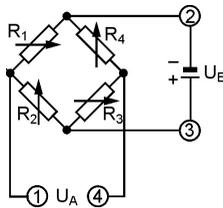


Fig. 2: Illustration for the rules of the signs in the Wheatstone bridge

Note 1-4: With reference to the strain gages themselves, we have:

point (1) positive, point (4) negative if $\epsilon_1 > \epsilon_2$ and/or $\epsilon_3 > \epsilon_4$.

point (1) negative, point (4) positive if $\epsilon_1 < \epsilon_2$ and/or $\epsilon_3 < \epsilon_4$.

Consequently, the size of ε_1 should be considered with respect to its effect on the resistance R. “Greater than” or “smaller than” must be used in an algebraic sense and not only for the absolute values, for example: $+10 \mu\text{m/m} > +5 \mu\text{m/m}$; $+2 \mu\text{m/m} > -20 \mu\text{m/m}$; $-5 \mu\text{m/m} > -50 \mu\text{m/m}$.

Note 1-5: Special features of strain measuring instruments

Here we would like to review the instruments commonly used in strain gage measurement technique. Equations (2) to (4) assume that a resistance variation in one or more arms of the bridge circuit produces a variation of the relative output voltage U_A/U_E . As a final result of the measurement this is only of limited importance. Since the actual strain value is more interesting, most of the special instruments have an indicator scale calibrated in “strain values”. The strain value $1 \mu\text{m/m} = 1 \cdot 10^{-6}\text{m/m}$ is used as the “Unit”, on older designs you may also find the designation “microstrain”.

All these special instruments are calibrated in such a manner that the indicated value ε^* is equivalent to the actual strain present, if there is only one active strain gage in bridge arm 1 (quarter bridge configuration, see Section 2) and if the gage factor k of the strain gage used corresponds to the calibration value of the instrument. The bridge arms 2, 3 and 4 are formed by resistors or by passive strain gages. This means, in fact, that $\varepsilon_2 = \varepsilon_3 = \varepsilon_4$ of equation (5) are zero and can be omitted. Some instruments are calibrated using a fixed gage factor of $k = 2$, others have a “gage factor selector” which can be set to the actual gage factor of the strain gage. Thus, if

$$k_{SG} = k_{instr.} \text{ then } \varepsilon^* = \varepsilon \quad (6)$$

If an instrument with a fixed calibration value $k = 2$ is used, the measured values ε^* must be corrected because the gage factors of the strain gages vary according to the grid material and grid configuration. The correction formula is

$$\varepsilon_1 = \varepsilon^* \cdot \frac{2}{k} \quad (7)$$

For all further considerations, these special features will be neglected since they are not necessary for a basic understanding of the bridge configuration. Further details may also be taken from the operating manuals of the instruments.

When strain measurements are taken, it is not the strain on the surface of the specimen that is of interest in most cases but instead the material stress.

Normal stress results in changes of length that can be measured with strain gages. The ratio of stress σ and strain ε in the elastic deformation range is defined by Hooke’s Law [3]:

$$\sigma = \varepsilon \cdot E$$

E expresses Young’s modulus (modulus of elasticity) which is a material parameter.

2 Elementary circuits with strain gages

The main fields of application of strain gages are experimental stress analysis and the design and manufacture of transducers. Depending on the measurement problem on hand, one or more strain gages are used at the actual measuring point. Designations such as “full bridge”, “half bridge” or “quarter bridge” indicate such arrangements, although actually these are not correct. In fact, the circuit used for the measurement is always complete and is either fully or partially formed by the strain gages on the specimen. It is then completed by fixed resistors, which are incorporated in the instruments (see fig. 3).

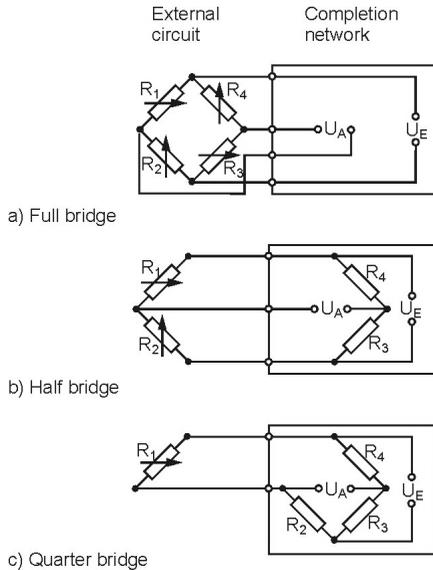


Fig. 3: Different versions of the Wheatstone bridge circuit

Transducers generally have to comply with more stringent accuracy requirements than measurements for stress analysis. Therefore, transducers should always have a full bridge circuit with active strain gages in all four arms.

Full bridge or half bridge circuits should also be used in stress analysis if different kinds of interferences need to be eliminated. An important condition is that cases of different stresses are

clearly distinguished, such as compressive or tensile stress, as well as bending, shear or torsional forces. (The compensation of interferences is covered in detail in Section 4.)

The mechanical interrelations are only treated as far as this is absolutely necessary. A detailed presentation can be found in [3].

2.1 Measurements on a tension bar

With a tension bar, a strain $\varepsilon_1 = \sigma/E$ in the direction of the force applied will occur. In the transverse direction, a transverse contraction $\varepsilon_2 = -\nu \cdot \varepsilon_1$ will occur. From this, a change of resistance $\Delta R_1 = \varepsilon_1 \cdot k \cdot R_1$ will be found in strain gage no. 1. For strain gage no. 2, this difference will be $\Delta R_2 = -\nu \cdot \varepsilon_1 \cdot k \cdot R_2$. Accordingly, this is also true for strain gages no. 3 and no. 4 respectively.

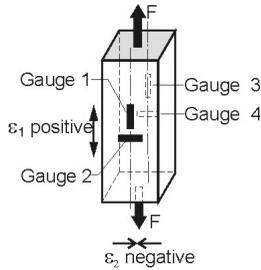


Fig. 4: The tension bar

Note 2-1: The factor ν , also called Poisson's Ratio, depends on the material and is valid in the material's elastic deformation range only. It has a value of around 0.3 for metals.

If all four strain gauges are connected in the sequence of their indices to form a full bridge (see fig. 3a) an output signal as described by equation (5) is produced

$$\frac{U_A}{U_E} = \frac{k}{4}(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4) = \frac{k}{4}[\varepsilon_1 - (-\varepsilon_1) + \varepsilon_3 - (-\nu\varepsilon_3)].$$

Introducing $\nu \approx 0.3$ and with $\varepsilon_1 = \varepsilon_3 = \varepsilon$ and $\varepsilon_2 = \varepsilon_4 \approx -0.3 \cdot \varepsilon$, the net signal will be

$$\frac{U_A}{U_E} = \frac{k}{4} \cdot 2,6 \cdot \varepsilon_1 \quad (8)$$

Note 2-2: A bridge circuit with four active strain gages gives a signal 2.6 times the value of the strain ε_1 in the tension bar's main direction of stress. Sometimes this factor is also called the bridge factor "B". Therefore, a general form of equation (8) is also

$$\frac{U_A}{U_E} = \frac{k}{4} \cdot B \cdot \varepsilon_1 \quad (9)$$

If compressive forces are to be measured, the opposite signs of the strains ε_1 to ε_4 are valid.

If the strain ε_1 in the main stress direction is unknown, this can be derived by transformation from equation (8):

$$\varepsilon_1 = \frac{4}{2,6 \cdot k} \cdot \frac{U_A}{U_E} \quad (10)$$

or, in the more general form of equation (9), transformed into

$$\varepsilon_1 = \frac{4}{B \cdot k} \cdot \frac{U_A}{U_E} \quad (11)$$

A half bridge circuit with only strain gages no. 1 and no. 2 as active strain gages gives only half the signal, because the factor B is only 1.3.

A combination of R_1 and R_3 formed by active strain gages and R_2 and R_4 with fixed resistors is also possible and gives $B = 2$. But this is without automatic compensation of the thermal expansion and other influences (see also Section 4). However, bending forces that are superimposed will be compensated.

2.2 Measurements on a bending beam

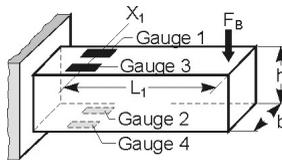


Fig. 5: The bending beam

The conditions in the case of the bending beam are a bit simpler and more favorable, see fig. 5. Here strain values are of the same absolute value but with the opposite sign. This means that strain

values on the upper side and on the underside of the beam are the same but with the opposite sign. We get particularly favorable conditions when adding the measured values according to equation (5):

$$\frac{U_A}{U_E} = \frac{k}{4} [\varepsilon_1 - (-\varepsilon_2) + \varepsilon_3 - (-\varepsilon_4)]; \text{ assuming } |\varepsilon_1| = |\varepsilon_2| = |\varepsilon_3| = |\varepsilon_4|$$

we have $B = 4$. With equation (9) again we have

$$\frac{U_A}{U_E} = \frac{k}{4} \cdot B \cdot |\varepsilon| = k \cdot |\varepsilon|$$

The absolute value of the strain is then $|\varepsilon| = \frac{U_A / U_E}{k}$

Its sign is then positive on the tensile side and negative on the compressive side.

If the directions of stress are not evident, then the signs can be derived from the rules given in Section 1.

If the bending beam can only be equipped with two active strain gages, the bridge arms no. 1 and no. 2 should be used. The bridge factor is then $B = 2$.

2.3 Measurements on a twisted shaft

The same favorable conditions as in the case of the bending beam are encountered in this case. The strain maxima are under $\pm 45^\circ$ with reference to the direction of shear plane (or under $\pm 45^\circ$ with reference to the shaft axis, which is in fact the same). The strain gages must be positioned as shown in fig. 6.

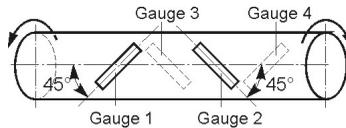


Fig. 6: Twisted shaft (SG = strain gage)

With clockwise torque, strain gages no. 1 and no. 3 will sense positive strain, strain gages no. 2 and no. 4 negative strain. The absolute values of all strains ε_1 to ε_4 are the same. Therefore, all strain values can be calculated according to equation (5). Using equation (9) and $B = 4$, the measured value is equal to

$$\frac{U_A}{U_E} = \frac{k}{4} \cdot B \cdot \varepsilon = k \cdot \varepsilon$$

From this we can derive the strain value as $|\varepsilon| = \frac{U_A / U_E}{k}$

If the sign of the torque is unknown, it can be found using the rules described in Section 1.

In the case of the twisted shaft, we can also use a half bridge with the active strain gages no. 1 and no. 2. The bridge factor will then be $B = 2$.

Note 2-3: If the measured signal must be transmitted from a rotating shaft via slip rings or similar devices (as is often required in torque measurements), the full bridge circuit should preferably be used. The advantage here lies in the fact that errors from the transmission path are considerably smaller than for a half or quarter bridge circuit (see Section 5).

3 Analysis and compensation of superimposed stresses, e.g. tension bar with bending moment

As discussed in Section 2, pure tensile or compressive forces as well as pure bending moments or torque cannot be assumed in all cases. In most practical cases, either two types or all three are combined. For example, this situation can be demonstrated for a beam under tensile force F and a bending moment M_B .

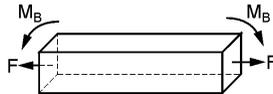


Fig. 7: Tension bar with bending moment superimposed

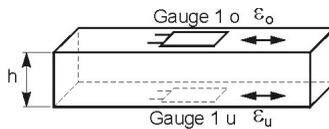


Fig. 8: Tension/bending beam according to fig. 7 with strain gauges applied

The following strain components can be detected on this beam:

- The total strain ϵ_o on the upper side of the beam and the total strain ϵ_u underneath the beam.
- The strain ϵ_N caused by the normal force (tensile or compressive).
- The strain component ϵ_B caused by the bending moment.

The composite strain values, ϵ_o and ϵ_u , can be measured with one strain gage each on the upper side and underneath the beam, orientated along the beam axis, see fig. 8.

Each strain gage has to be inserted into a quarter bridge (see fig. 3c) separately and its measured value must be recorded. The strain gage on the upper side delivers the sum $\epsilon_o = \epsilon_N + \epsilon_B$, the strain gage underneath the difference $\epsilon_u = \epsilon_N - \epsilon_B$. The strain components can be computed from these two values with the help of the relationship

$$\varepsilon_N = \frac{1}{2}(\varepsilon_O + \varepsilon_U) \quad (12)$$

$$\varepsilon_B = \frac{1}{2}(\varepsilon_O - \varepsilon_U) \quad (13)$$

A graphic solution with three examples is given in fig. 9.

If compensation of the thermal expansion of the specimen is required, strain gages should be used whose response to temperature has been adapted to the component material's thermal expansion (see Note 4-2). If these are not available or if the compensated range is insufficient, both sides of the specimen should be equipped with one longitudinal and one transverse strain gage (see fig. 10). Each side must then form a separate half bridge (see fig. 3b). The indicated values will be (with ν = Poisson's ratio):

$$\varepsilon_O^* = (1 + \nu) \cdot \varepsilon_O = (1 + \nu)(\varepsilon_N + \varepsilon_B) \quad \varepsilon_U^* = (1 + \nu) \cdot \varepsilon_U = (1 + \nu)(\varepsilon_N - \varepsilon_B)$$

(see also Section 2.1). The result can be analyzed with the help of equations 12 and 13 or found by using graphic methods as shown in fig. 9. The indicated values ε_O^* and ε_U^* should first be divided by $(1 + \nu)$. Temperature compensation methods are discussed in Section 4.

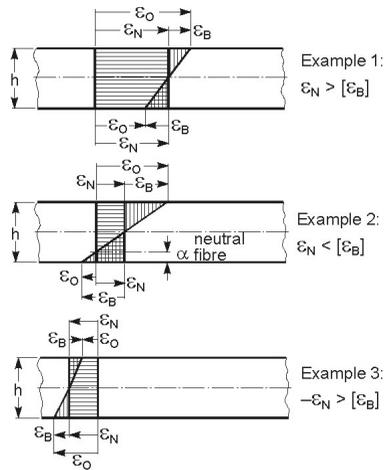


Fig. 9: Examples of graphical solutions for mixed stresses.

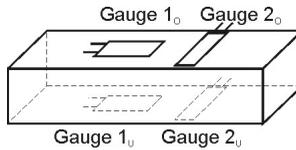


Fig. 10: Tension /bending beam according to fig. 7 fitted with longitudinal and transverse strain gages

When designing a force transducer, the aim is always to measure the forces in the direction of the transducer axis only instead of the bending strains resulting from eccentric or side loads, etc.

In this case, a combination of strain gages should be used in accordance with fig. 4. Strain gages must be symmetrical both with regard to their position and specification. All four strain gages form a complete Wheatstone bridge circuit, which is optimum for electrical symmetry.

The absolute strain values (or the proportional changes in strain gage resistance) are **subtracted** if they appear with **opposite signs in opposite bridge arms**. It is obvious that the signals resulting from the normal force are added (see also fig. 4) and signals from the bending force compensate one another. The components from the bending force are distributed as follows:

$$\text{SG no. 1: } +\epsilon_B \qquad \text{SG no. 2: } -\nu \cdot \epsilon_B \qquad \text{SG no. 3: } -\epsilon_B \qquad \text{SG no. 4: } +\nu \cdot \epsilon_B$$

The absolute values of the corresponding signals are equal, which leads to complete compensation (see also Section 4).

This principle can also be applied to compensate for axial forces in the bending beam or bending forces in the twisted shaft (see also Sections 2.2 and 2.3).

Note 3-1: Generally speaking, the four strain gages of such a transducer could also be connected in this way into a half bridge, by series connecting strain gages no. 1 and no. 3 and strain gages no. 2 and no. 4 into one bridge arm each, as illustrated in fig. 11.

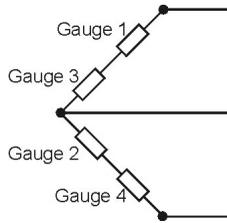


Fig. 11: Half bridge with four strain gages

This circuit is symmetrical with respect to the position of the strain gages but not with respect to the connection leads within the overall bridge circuit, which must be completed with the complementary half bridge located inside the instrument. In addition, the sensitivity is only half the sensitivity of the full bridge. The measured values of series-connected strain gages will certainly not add up. The following relationship is true for the circuit shown in fig. 11:

$$\frac{U_A}{U_E} = \frac{1}{4} \left(\frac{\Delta R_1 + \Delta R_3}{R_1 + R_3} - \frac{\Delta R_2 + \Delta R_4}{R_2 + R_4} \right) \quad (14)$$

From this, it can easily be seen that compared to equation (4) we get only half the signal.

4 Compensation of interference effects, especially temperature effects

Equation (4) provides a basic understanding of the principle of the Wheatstone bridge circuit. It will also lead the way to a compensation of unwanted changes of the strain gage resistance caused by interference effects.

Changes of the strain gage resistance **of the same sign** appearing in **neighboring** arms will be **subtracted** with respect to the bridge output signal. If we manage to position the strain gages on the specimen in such a way that interferences will influence neighboring strain gages in the same way, they compensate one another. The degree of the compensation depends on the uniformity acquired.

Note 4-1: The following interference effects are considered: temperature, humidity, hydrostatic pressure, vacuum, magnetic fields, radiation, etc. The degree of their respective influence is quite different and is also dependent on the strain gage construction and material. It is always recommended that, as far as possible, strain gages are shielded by suitable means and to compensate the remaining interference effects are compensated.

The principle of compensation is to be illustrated by the example of temperature effects.

We can assume, without considering the reasons in detail, that a bonded strain gage can have different resistance values, if the temperature of the specimen changes. This may be caused, for example, by the thermal expansion of the specimen, which is of course also reflected by the strain gage.

If this temperature variation occurs during the mechanical loading of the structural part, we will get a composite strain signal. One component is the desired effect from the mechanical stress, ε_M and the other component is the undesired effect from the thermal expansion, ε_W .

$$\frac{\Delta R}{R_0} = k \cdot (\varepsilon_M + \varepsilon_W). \quad (15)$$

If four strain gages are applied to the specimen and connected into a full bridge as described in the examples in sections 2.1, 2.2 and 2.3, the following situation is found:

SG no.	ε_M	ε_W
1	Positive	Positive
2	Negative	Positive
3	Positive	Positive
4	Negative	Positive

We can see that the thermal component in the total strain ε_W has the same sign for all strain gages, provided they are all subjected to the same change in temperature. Based on equation (5) we find the expression

$$\frac{U_A}{U_E} = \frac{k}{4} [(\varepsilon_M + \varepsilon_W)_1 - (-\varepsilon_M + \varepsilon_W)_2 + (\varepsilon_M + \varepsilon_W)_3 - (-\varepsilon_M + \varepsilon_W)_4] = \frac{k}{4} \cdot 4\varepsilon_M$$

The interference effect ε_W has been compensated.

The same method of compensation can be used for a half bridge.

This method of compensation cannot be implemented with the **quarter bridge**, which is widely used in experimental stress analysis. The only way to overcome this is to use a “compensating strain gage” to expand the quarter bridge into a half bridge. It is very important to select and apply the compensating strain gage correctly in order to achieve the correct result. The compensating strain gage:

- a) must have the same physical properties as the “active” strain gage
- b) must be applied in a spot where it will be subjected to the same interference effect as the active strain gage.
- c) must only be subjected to the interference effect and never to the quantity to be measured ε_M or its side effects.

If a suitable spot cannot be found on the specimen itself, an extra piece of material can be used. The compensating strain gage can be bonded to it and the arrangement can then be placed near the active strain gage. This will lead to a proper compensation of the interference effects (of temperature) without influences from the stress, since there is no mechanical coupling. The piece with the compensating strain gage should be made from the same material as the original specimen with the active strain gage, especially if the interference effect will lead to deformation of the material, such as thermal expansion, compression by hydrostatic pressure, magnetostriction, etc.

Note 4-2: The temperature response of a strain gage can be adjusted so that the temperature effect on the material can be compensated – within technical feasibility. Therefore, each HBM strain gage (exception: LD20 high strain gage) will be adapted to the thermal expansion coefficient of a certain material. This adaptation considerably reduces the error caused by temperature.

The remaining temperature response is specified on the strain gage package so that the temperature error can be corrected mathematically.

5 Compensation of the influence of lead resistances

Conductors form a resistance for the electrical current, depending on the length of the conductor and its cross sectional area. Cable resistances vary noticeably with the ambient temperature due to the large temperature coefficient of copper: ($\alpha_{RCu} \approx 0.04/^\circ C$).

As a consequence, cables should always be kept as short as possible and should have a large cross section, because changes of cable resistance are smaller for smaller resistance values. There are, of course, natural limits.

Considering the different connection methods shown in fig. 12, we find two main differences: As long as a full bridge circuit can be used, the connection cables that are to be considered are **not incorporated** in the bridge circuit. For all other configurations, the cable is always a **part** of the bridge circuit (completed by resistances in the instrument).

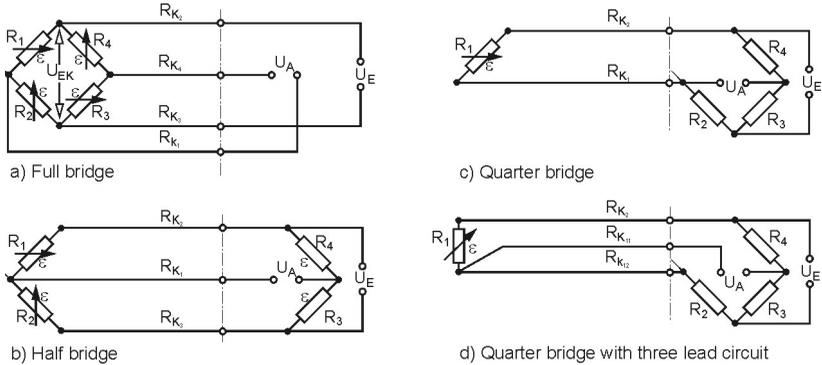


Fig. 12: Connections between the strain gages and the completion elements of the amplifier

a) Full bridge circuit, fig. 12a

For the full bridge, it can in general be assumed that the connections **within** the bridge are short and their interference effects are negligible. The cables R_{K2} and R_{K3} can be considered as a lumped resistance in the bridge excitation circuit. The actual bridge will then be supplied with a (smaller) voltage U_{EK} according to the voltage drop across the cable resistance.

$$U_{EK} = U_E \cdot \frac{R_B}{R_B + R_{K2} + R_{K3}} \quad (16)$$

From this we can derive:

$$\frac{U_A}{U_E} = \frac{k}{4} (\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4) \cdot \frac{R_B}{R_B + R_{K2} + R_{K3}} \quad (16a)$$

In equation 16a, R_B is the bridge shunt resistance between points 2 and 3 (see also fig. 1).

$$R_B = \frac{(R_1 + R_2)(R_3 + R_4)}{R_1 + R_2 + R_3 + R_4} \quad (17)$$

R_{K2} , R_{K3} are the resistances of the cables K_2 and K_3 (their variation due to temperature can normally be disregarded).

Note 5-1: The sensitivity of a bridge with $R_B = 120$ Ohms will be reduced by, say, 5.8 % when using a special cable with a length of 100 meters (i.e. 2×100 m core length) and a cross section of 0.5 mm^2 . For $R_B = 600$ Ohms, this decrease in sensitivity will be only 1.2 %.

Both cables R_{K1} and R_{K4} are normally connected in series with the high input impedance of the measuring amplifier, or in series with the almost infinite impedance of a (properly balanced) zero balancing instrument or compensator. Therefore, these cables do not influence the result of the measurement.

b) Half bridge circuit, fig. 12b

In the case of the half bridge, the cable R_{K2} is connected in series with strain gage No. 1 and R_{K3} in series with strain gage No. 2. This will affect the overall sensitivity of the strain gage and could also be considered as an apparent change in the gage factor of both strain gages. The apparent gage factor k^* is therefore

$$k^* = k \cdot \frac{R_{DMS}}{R_{DMS} + R_K} \quad (18)$$

This change in sensitivity could also be applied to the relative bridge output voltage as follows

$$\frac{U_A}{U_E} = \frac{k}{4} \left(\varepsilon_1 \cdot \frac{R_1}{R_1 + R_{K2}} - \varepsilon_2 \cdot \frac{R_2}{R_2 + R_{K3}} \right) \quad (19)$$

With $R_1 = R_2 = R$ and $R_{K2} = R_{K3} = R_K$ we will have

$$\frac{U_A}{U_E} = \frac{k}{4} (\varepsilon_1 - \varepsilon_2) \frac{R}{R + R_K} \quad (20)$$

Every variation of the cable resistance being connected in series with the strain gage will have the same effect as the change of the strain gage resistance itself. In a half bridge circuit, temperature effects on cables can be compensated.

The method is described in section 4. A symmetric configuration of both bridge arms K_2 and K_3 must be chosen. The cables must be made from the same material, have the same length and cross section, and be subject to the same ambient temperature distribution along their length.

The cable K_1 has no influence on the results, as already discussed in the case of a) the full bridge.

c) Quarter bridge circuit, fig. 12c

Because of its high sensitivity to interferences, the pure quarter bridge circuit should only be used under the most favorable conditions or if all interference effects are well known and can be compensated appropriately. Besides, there is a loss of sensitivity due to the resistance in the connection cables:

$$k^* = k \cdot \frac{R_{DMS}}{R_{DMS} + R_{K1} + R_{K2}} \quad (21)$$

or with reference to the relative bridge output voltage:

$$\frac{U_A}{U_E} = \frac{k}{4} \cdot \frac{R_1}{R_1 + R_{K1} + R_{K2}} \quad (22)$$

The quarter bridge is only recommended for measurements under stable temperatures or if temperature compensated strain gages are used (see also note 4-2). The proper functioning of the temperature compensation can only be achieved with the so-called “three wire circuit” shown in fig. 12d. This method requires that cables K_2 and K_{12} be of the same material, same resistance, same length and subject to the same temperature influences, while cable K_{11} is not critical in this respect. The special trick of this circuit is that the cable K_2 is connected in series with the strain gage No. 1, whereas K_{12} is part of the neighboring bridge arm and is in series with the complementary resistor R_2 . This provides the same compensating conditions as for the half bridge circuit.

For the loss of sensitivity, the strain gage equation (18) is valid or

$$\frac{U_A}{U_E} = \frac{k}{4} \cdot \varepsilon_1 \cdot \frac{R_1}{R_1 + R_{K2}} \quad (23)$$

6 Table on different circuit configurations

The table below* shows the dependence of the geometrical position of the strain gages, the type of bridge circuit used and the resulting bridge factor B for normal forces, bending moments, torques and temperatures. The small tables given for each example specify the bridge factor B for each type of influencing quantity. The equations are used to calculate the effective strain from the bridge output signal U_A/U_E . If strain measuring devices are used, equations (7) or (11) can be applied accordingly.

<p>1</p>	$\varepsilon = \varepsilon_n + \varepsilon_b = \frac{4}{k} \cdot \frac{U_A}{U_E} - \varepsilon_s$ <table border="1"> <thead> <tr> <th>T</th> <th>F_N</th> <th>M_b</th> <th>M_d</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>1</td> <td>1</td> <td>0</td> </tr> </tbody> </table>	T	F _N	M _b	M _d	1	1	1	0	<p>9</p> $\varepsilon = \varepsilon_n = \frac{1}{2(1+\mu)} \cdot \frac{4}{k} \cdot \frac{U_A}{U_E}$ <table border="1"> <thead> <tr> <th>T</th> <th>F_N</th> <th>M_b</th> <th>M_d</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>2(1+ν)</td> <td>0</td> <td>0</td> </tr> </tbody> </table>	T	F _N	M _b	M _d	0	2(1+ν)	0	0		
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Note: A cylindrical shaft is assumed for torque measurement, examples 13, 14, 15. For reasons of symmetry, bending in X and Y direction is allowed. The same conditions also apply for the bar with square or rectangular cross section. Explanation of symbols:

T = temperature

F_N = longitudinal = normal force

M_b = bending moment

M_{bX}, M_{bY} = bending moment for X and Y directions

M_d = torque

These influences result in strains ε_s, ε_n, ε_b, ε_{bX}, ε_{bY}, ε_d respectively.

ε = the effective strain at the point of measurement Z₀ for P and M_b in normal direction, for M_d under φ = + 45°

ν = Poisson's ratio

= active strain gage

= strain gage for temperature compensation

= passive strain gage or resistor

*Table compiled by Mr. J. Bomhardt, first published in ELEKTRONIK 16 (1967) No. 19.

7 Some remarks on the limits of the compensation of interference effects

The explanations of the compensation methods for interference effects by means of the Wheatstone bridge circuit require some additional comments:

The compensation of the temperature expansion is only correct if some conditions are strictly fulfilled. These include:

- a) symmetry of the bridge,
- b) identical temperature coefficients of all materials used,
- c) identical resistances of all parts in the bridge arms which are combined for compensation,
- d) identical temperatures on all compensating elements in the bridge circuit,
- e) identical active grid areas. The bending radii and grid directions of strain gages located on curved surfaces should also be the same.

In all practical applications, there will always be smaller or bigger tolerances, therefore remaining errors have to be anticipated even if compensation methods are applied. High-precision transducers are specially treated for compensation of the remaining errors.

The possible compensation of mechanical interference effects is also limited, for example, the compensation of the bending force on a tension bar. The main reasons are:

- a) tolerances in the alignment of the strain gages on the specimen,
- b) sensitivity tolerances between the strain gages,
- c) modulus of elasticity being dependent on the direction of force,
- d) dimensional (manufacturing) tolerances of the specimen,
- e) deformation of the cross section of a body under stress which results in imperfections of the previous symmetry,
- f) anisotropic effects of the elastic properties of the material of the specimen.

From experience we can learn that any interference effect can be compensated to, say, 95 to 98% provided the strain gage is applied properly. The remaining approximately 2 to 5% is still contained in the measurement result.

These limits should be kept in mind when discussing the compensation of interference effects.

8 The linearity error of the Wheatstone bridge circuit

In strain gage techniques, a linear relationship between the relative change of the strain gage resistance and the relative output voltage of the Wheatstone bridge, as described by equation (4), is usually assumed. This is, of course, applicable for all practical purposes, as long as all restrictions for equation (4) are adhered to. But if the relative changes of the strain gage resistance become greater, say a few percent, or if extreme accuracies are required, the assumptions can no longer be made. The actual relationship results from equation (2) or from the following derived equation

$$\frac{U_A}{U_E} = \frac{R_1 + \Delta R_1}{R_1 + \Delta R_1 + R_2 + \Delta R_2} - \frac{R_4 + \Delta R_4}{R_3 + \Delta R_3 + R_4 + \Delta R_4} \quad (24)$$

The size of the linearity error depends, on the one hand, on the absolute value of the relative deviation of the resistance and its sign, and, on the other hand, on the bridge arms where the change occurs. The usual configurations, i.e. quarter, half and full bridge (see section 2) are dealt with separately in the following sections. Linearity deviations are calculated and plotted as an error curve with reference to the tangent through zero. The tangent through zero is described by equation (4). In general, the error curve describes the deviation of the bridge output as a function of the input signal.

Take the calculated deviations with the opposite sign to correct a measured value.

8.1 Quarter bridge

The linearity error of the quarter bridge is first investigated because this is the most often used and simplest strain gage circuit (see fig. 3c).

Assuming $R_1 = R_2 = R_3 = R_4 = R$, from equation (24) we can derive

$$\frac{U_A^*}{U_E} = \frac{\Delta R_1}{2(2R + \Delta R_1)} \quad (25)$$

The tangent through zero can be calculated from equation (4) or from the special quarter bridge form

$$\frac{U_A}{U_E} = \frac{\Delta R_1}{4R} \quad (26)$$

Defining the relative linearity error as

$$F_{Lrel} = \frac{\frac{U_A^*}{U_E} - \frac{U_A}{U_E}}{\frac{U_A}{U_E}} \cdot 100\% \quad (27)$$

we find in this case:

$$F_{Lrel} = \frac{-\Delta R}{2R + \Delta R} \cdot 100\% \quad (28a)$$

or as an approximation

$$F_{Lrel} \approx \frac{-\Delta R}{2R} \cdot 100\% \quad (28b)$$

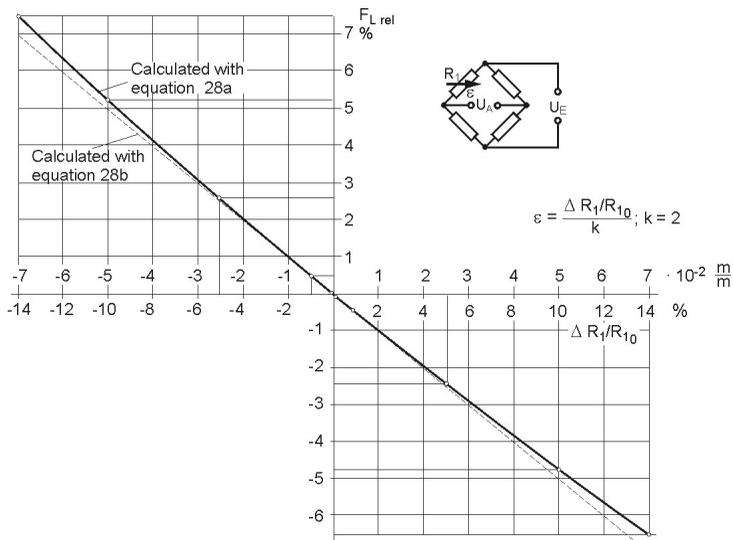


Fig. 13: Relative linearity error of the quarter bridge as a function of the relative change of resistance or of the strain

Note 8-1: In diagrams 13 and 14, it becomes quite obvious that percentages are always difficult. Abscissa and ordinate both have a percentage scale but with different orders of magnitude: The linearity error is a

percentage of the relative change of the resistance, which is also a percentage itself. In a mathematical sense, the linearity error has the magnitude of 10^{-4} . (In this context, refer to the VDI/VDE 2635 guideline.)

Note 8-2: Because $\Delta R/R = k \cdot \varepsilon$ from equation (1) we can take $F_{Lrel} \approx -\varepsilon$ as rule of thumb for $k = 2$.

8.2 Half bridge

In a half bridge circuit (see also fig. 3b), active strain gages are used in both bridge arms (R_1 and R_2).

Note 8-3: In this paper, where R_2 is only used for temperature compensation or for the compensation of other interference effects, all applications are considered as quarter bridges.

Assuming again for the sake of clarity that $R_1 = R_2 = R_3 = R_4 = R$, we find from equation (24):

$$\frac{U_A^*}{U_E} = \frac{-\Delta R_1 - \Delta R_2}{2(2R + \Delta R_1 + \Delta R_2)} \quad (29)$$

Two characteristic applications of the half bridge are described in section 2.1 and 2.2 or 2.3 respectively. It is essential for such applications that positive and negative strain occurs at the same time, which is then measured by strain gages connected as neighboring arms of the bridge. In these examples, strain gage no. 1 is subject to positive strain and strain gage no. 2 to negative strain. Therefore, we have positive values for ΔR_1 and negative values for ΔR_2 . Equation (29) is in the first instance a confirmation of the statement of section 1 that measured values with opposite signs in neighboring arms of the bridge are added, but the values ΔR_1 , ΔR_2 are subtracted as they are responsible for the linearity error in the denominator of equation (29). They disappear completely if they have the same absolute value. This is true for the bending beam (see section 2.2) and for the twisted shaft (see section 2.3). Measuring circuits of this kind do not have inherent linearity errors! For the tension bar (see section 2.1), there is an inherent linearity error because of $\varepsilon_1 : \varepsilon_2 = \Delta R_1 : \Delta R_2$ with $\Delta R_2 = -\mu \cdot \Delta R_1$ which is smaller than for the quarter bridge:

$$F_{rel} = \frac{-(\Delta R_1 + \Delta R_2)}{2R + \Delta R_1 + \Delta R_2} \cdot 100\% \quad (30a)$$

or as an approximation

$$F_{Lrel} \approx \frac{-(\Delta R_1 + \Delta R_2)}{2R} \cdot 100\% \quad (30b)$$

Note 8-4: Because $\Delta R/R = k \cdot \varepsilon$ in equation (1) we can take : $F_{Lrel} \approx -1 \cdot (\varepsilon_1 + \varepsilon_2)$ as rule of thumb.

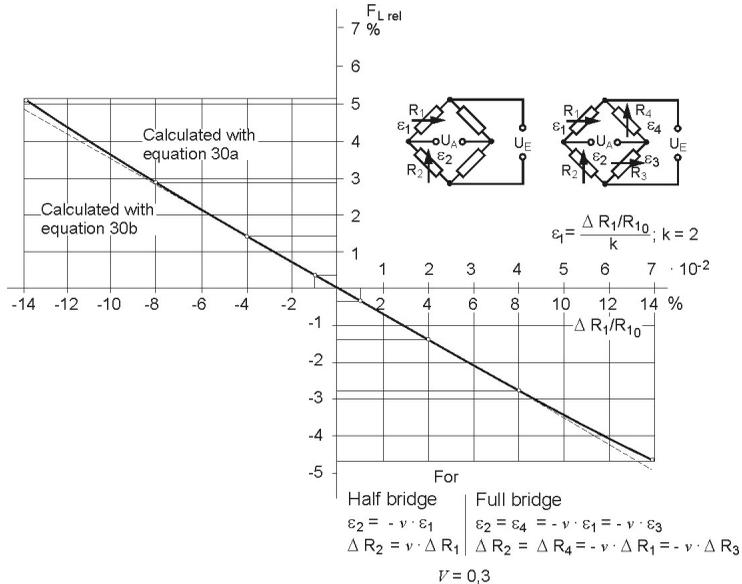


Fig. 14: Relative linearity error of the half bridge circuit as a function of the change of the resistors R_1 and R_2 or R_1 to R_4 . Valid for $\Delta R_{1,3} \sim \varepsilon_{1,3}$ and $\Delta R_{2,4} \sim \varepsilon_{2,4} = -\nu \cdot \varepsilon_{1,3}$.

8.3 Full bridge

In a full bridge circuit (see also fig. 3a), active strain gages are used in all four bridge arms R_1 to R_4 . Assuming $R_1 = R_2 = R_3 = R_4 = R$, we find from equation (24)

$$\frac{R + \Delta R_1}{2R + \Delta R_1 + \Delta R_2} - \frac{R + \Delta R_4}{2R + \Delta R_3 + \Delta R_4} \quad (31)$$

Using the types of stress and the corresponding circuits described in sections 2.2 and 2.3, there will be no inherent linearity errors. In this respect, full bridge and half bridge are equivalent.

Taking the example of the tension bar of section 2.1 with the strains $\varepsilon_1 = \varepsilon_3$; $\varepsilon_2 = \varepsilon_4 = -\nu \cdot \varepsilon_1$ and the corresponding proportional changes of resistance $\Delta R_1 = \Delta R_3$; $\Delta R_2 = \Delta R_4 = -\nu \cdot \Delta R_1$, we have a remaining linearity error identical to that of the half bridge. Therefore, fig. 14 is also true for the full bridge.

9 Bibliography

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Hottinger Baldwin Messtechnik GmbH

P. O. B. 100151 · D-64201 Darmstadt

Im Tiefen See 45 · D-64293 Darmstadt

Tel.: +49 6151 803 0 · Fax: +49 6151 803 9100

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